Regrasp Planning for Polygonal and Polyhedral Objects

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May 11, 2007

1 Introduction

When we look at our hands and think about what they can do, grasping is mostly in our imagination. We can easily grasp an object without calculating proper grasping positions according to the model of the object. But this ability of humans cannot be transferred to a machine at ease. In Robotics, grasping is an operation performed by a robot hand for secure holding an object. We know some practical outcomes such as parallel jaw grippers which usually applied for pick and place operation in factories. They can work properly because of certain industrial or factory settings. In such environments, we exactly know what kinds of objects there are, and their geometries are precisely known. The precise geometry of the robot arm is also known. Thus, a planner can precalculate *fixed* contact positions and controls of the robot arm's joints to perform a task without uncertainty. Let us deeply consider the definition of grasping. A grasp achieves stability if and only if contact positions are strictly fixed even when any disturbance is exerted on the object. Object manipulations without impact between the contact of the fingers and the object, which can be applied in this situation, are limited to be rotating wrist, elbow, arm or other joints unrelated with the contact positions. These processes seem to be unnatural comparing with what a human does. Beyond these awkward operations, the capability of a human hand is not only limited for grasping. Dexterity of a hand can be exploited for many tasks in real applications. For example, we can rotate an assembly part using our fingers then assemble without releasing it. We can tie a shoelace by using a little times to practice. Interestingly, after nearly half a century of robotics research there is no robot that can efficiently perform any of these ordinary tasks. The operation discussed above that a hand changes grasping configuration, is formally called *dexterous manipulation.*

There seem to be multiple source of difficulty to perform dexterous manipulation. It relies on various and broad knowledge in robotics. This work decomposes the system of dexterous into five main levels as shown in Table 1. Each level has its own difficulties and complexities challenging many researchers to overwhelm. The purpose of this decomposition is to decrease the complexity of dexterous manipulation system by separating task and mechanical constraints. The highest level is task planning which determines a proper task for the robot. The main objective considered in this level is to generate the motion of the object to achieve a desired configuration or determine a proper grasp which can accomplish a task with regardless the mechanical constraints. Planning contact positions and motions of the fingers is performed in the second level which is mainly focused in this work. Moreover, in this level, analyzing the grasp and choosing an optimal set of grasps are regarded as well. Regrasp planning is necessary for dexterous manipulation as a link between task and constraints. A task planner provides desired grasping configurations to achieve a task. The function of a regrasp planner is to compute trajectories meeting grasp stability constraints from the current grasping configuration to the desired configurations. This relaxes the complexity and dependency between task requirements and constraints in the lower levels. In real applications, a robot finger is not a point and the model of a robot hand is specified. Motions of the fingers have to satisfy more constraints which are kinematics constraints and dynamics constraints. Kinematics constraints relate with the robots workspace, joint limits, accessability, contact constraints, collision constraints, etc. Moreover, regrasping is not a static operation. The fingers change their positions. More realistically, dynamic constraints are considered such as sliding or rolling a finger along the objects surface which can be formulated into relationships between joint space and force space. The lowest level is the implementation of a robot hand which induces kinematics constraints, dynamics constraints, workspace, joint torque limitation, etc. used in the third and forth levels.

Table 1: Levels of Dexterous manipulation

Task planning
Regrasp Planning
Kinematics
Dynamics
Robot hand

This work studies the problem of regrasp planning which computes a sequence of finger repositioning from initial grasping configuration to a desired configuration for polyhedral-modelled object and a set of discrete points based on the following assumptions. The hands are assumed to be equipped with four and five finger in two and three dimensions, respectively. Three-fingered grasp is sufficient for grasping a two-dimensional object and four fingers is sufficient for three-dimensional grasp. The other one is used to switch grasping position. Our planner aims to construct general solution satisfying grasping constraints regardless task constraints, kinematic constraints, dynamic constraints, etc. The most advantage of general solution is independence. It is applicable with any task or hand in the real world. A finger is therefore assumed to be a free-flying point contact. To maintain stability, grasping constraint considered in this work is associated with force closure property. Every grasping configuration in the obtained sequence of finger repositioning has to satisfy force closure property to ensure stability during the entire repositioning process.

The main contribution of this work is to proposed a framework for regrasp planning problem. Our planner reports a general set of feasible finger repositioning satisfying force closure property for task and constraint independence. An approach using a structure called *Switching Graph* has been introduced. Connectivity in a graph presents ability to change a grasping configuration to another. This allows the regrasp planning to be transformed to graph search. A node in switching grasp represents a connected set of force closure grasps for given surfaces. Any grasps of which representations are in the same node can be transformed to one another using finger sliding along the continuous surfaces. An edge connecting two nodes indicates the ability of switching one finger to another different surface. Based on this structure, the obtained results are not a single solution, they are a set of feasible solutions. An advantage of a set of solutions is that it allows any planner to find a sequence of grasping positions which optimized according to some considered criteria or to add more constraints for practical uses. In contrast, a node in switching graph for a set of contact points represents a three(four)-fingered grasp in two(three) dimensions. There exist an edge joining two nodes when two grasps associated these nodes have one distinct grasping point. This can be done without the use of local surface information and only finger switching process can be performed. Actually, an object has its own model. Local surface information can be exploit for finger sliding or rolling along the surface to change grasping configuration. A model approximation is applied to estimate the model of object, which allows our planner to acquire grasping positions and normals for calculating a feasible trajectory.

2 Related Works

Regrasp problem consists of various problems in many subfields on robotics. Firstly, we have to define what we want the robot to do. This is according to task constraints. Based on the classification of grasp by Cutosky [18], two main grasping types are concerned, fingertip grasp and power grasp. Fingertip grasps achieve dexterity by holding the objects by the tips of the fingers. Power grasps are distinguished by large areas of contact between the object and the fingers and palm which do not allow the motion of the grasped object. The grasps perform with low dexterity. For regrasping, the fingertip grasps are preferred since the problem required dexterity of grasps.

Regrasp planning is the main theme of this work. The method reports a sequence of fingers' position from initial grasp to desired grasp given by task planner. The obtained grasping positions not only associate with task requirement but also satisfy stability constraint. The force closure property is applied to satisfy the stability constraint. This means that every grasping position in a sequence calculated by the method has to achieve force closure grasp.

In practice, a robot finger is not a point. A grasp has to satisfy kinematic constraints and dynamic constraints as well. Motions of fingers when a regrasp process is performed, also introduce to a manipulation planning problem which mainly mentions accessability and collision avoidance of a path from an initial configuration to a desired one.

2.1 Robot Hands

Dexterous manipulation or regrasp problem require a manipulator which is able to change a grasped object's configuration with respect to the hand without releasing it. The robot hand is one suitable manipulator for this task. It may be designed to be an approximation of the human hand or specified for particular tasks. A well-known three-fingered robot hand is Barret Hand [68] commercially made by Barrett Technology Inc. Two fingers can be spread synchronously by 180° around the palm. The Utah/MIT hand [27] is the first anthropomorphic hand with four fingers. Each finger has four degrees of freedom. The whole hand system is very large including the out-hand actuators. The Robonaut hand [37] designed for space based operations has five fingers. The hand combined with wrist and forearm has fourteen degrees of freedom. Another anthropomorphic hand is the DLR-Hand [7]. The hand consists of four fingers with the actuators embedded inside.

2.2 Contact Kinematics, Dynamic and Control of Manipulation

When the object has been grasped, the hand is possible to perform in-hand manipulation. To gain more dexterity, the hand is not required to maintain a rigid grasp. It may therefore roll, slide or release and place fingers to change the grasp configuration. The accurate control of the force applied to the object, which associates with the contact constraints is required to achieve the operations.

One approach that the dexterous hand manipulates an object, is exploiting a rolling contact. Rolling is the operation that the fingertip rolls without slipping on the object's surface. It is defined by the constraints that the fingertip and object velocities are equal at contact point. The kinematic constraints and transformations between task-space and local coordinates are presented in [29] and [42]. The rolling constraints are formulated in different ways. Kerr and Rott [29] derived the force analysis for the systems using a set of differential equations to describe the motion of the object with pure rolling contact. Montana [42] proposed a method for relating relative rigid body motion to the rates of change of contact coordinates using a matrix formulation of the motion of a point of contact over the rolling surfaces. Sarkar *et al.* [62] introduced local contact coordinates which allow them to formulate the dynamics and control of manipulation via rolling contacts in explicit equations relating the velocities and accelerations of the contact points. The formulation admits motion of the contacts during the manipulation process. Li *et al.* [33] developed a unified formulation describing the relationship between the object motion and the joint motion.

Dexterous manipulation sometimes exploits slippage between the fingers and the object to change grasping configuration. Sliding a finger along the surface of an object requires a good model of the contact friction which is mostly assumed Coulomb friction model. A finger exerts a force inward to the object's surface when it slides along the surface. According to Coulomb friction model, when the finger is sliding, the contact force must lie on the edge of the friction cone. Brock [6] derived a kinematic relation between the object motion, the motion constraints and the grasp forces. Cole *et al.* [13] presented a coordinated control law for sliding contacts between an object and fingertip including a problem of choosing contact positions for collision avoidance. In [14], the sliding motion of the fingertips along the object's surface is dynamically controlled simultaneously with controlling the position and orientation of the held object. Zheng *et al.* [75] formulated a dynamic control of a three-fingered robot hand manipulating an object in three dimensional space. One finger is allowed to slide on the object's surface. Motion equations of the whole system are derived. They also proposed a dynamic control law for linearizing the system dynamics and realizing the desired object motion, the desired finger sliding and desired grasping force.

Combinations of rolling and sliding are in consideration as well. Cai and Roth [8], [9] studied spatial motions combining rolling and sliding between rigid bodies for point contact and line contact, respectively. Chong [12] proposed an algorithm generating finite motion of object by considering sliding contacts as well as rolling contacts between the fingertips and the object. The minimum contact forces and minimum joint velocities are solved for the relative velocity at the contact point.

Forces applied to the object by the fingers are controlled for the desired manipulation. Kerr and Roth [29] developed a hand Jacobian which calculates the joint torques from the desired contact forces. Yoshikawa and Nagai [72] decomposed forces into two components. Manipulating or external forces produce a net force and torque on the object. The other forces are grasping or internal forces which produce no net force nor torque on the object. These forces are used to maintain a secure grasp.

The same authors gave a physically reasonable definition of manipulating force and grasping force for two-, three- and four-fingered hands in [73]. They also presented an algorithm for decomposing a given fingertip force into manipulating and grasping forces. Using the concept of the manipulating and grasping forces, they proposed a dynamic manipulation/grasping controller of multifingered robot hands based on the dynamic control and the hybrid position/force control. The controller consists of a compensator which linearizes the whole grasping system and a servo controller for the linearized system. Nakamura *et al.* [44] discussed the dynamical coordination of a multifingered robot hand. The coordination problem is solved in two phases. Firstly, determine the resultant force used for maintaining dynamic equilibrium and for generating the restoring force. Secondly, determine the internal force used to satisfy the static frictional constraints and is related to contact stability. Li *et al.* [34] studied a formulation of dynamic stability of grasping using Lapunov stability theory for measurement purpose.

The systems discussed above are formed by complex constraints. A system that a manipulation is achieved by low velocity motions is called quasi-static. Quasi-static analysis results are therefore much simpler and practical. Fearing [22] considered slip from a quasi-static viewpoint to achieve grasp stability. Yoshikawa *et al.* [74] used controlled slip in quasi-static system to modify the grasp and increase manipulation range for a three-fingered robot hand.

2.3 Grasp Definition

Secure holding an object in a robot hand is required in grasping. The concept of a firm grasp is formalized in various ways. Equilibrium, force closure and form closure property are usually applied to ensure the stability of a grasp. Equilibrium grasp is a grasp that the resultant of forces and torques exerted to the grasped object are zero. According to the definition, an equilibrium grasp cannot resist any disturbance. This property is therefore not sufficient to ensure the stability of a grasp. Force closure grasp is a grasp that can exert a resisting force and torque balancing any external disturbance on the object. A closely related property to force closure is form closure firstly investigated by Reuleaux [58]. The distinction between form closure and force closure is that form closure considers the immobility of an object in presence of fixed contact points whereas force closure considers how contact points can exert force and torque on an object. Another difference between form and force closure is the presence of friction. Friction effect is considered in force closure while it is neglected in form closure analysis. Markenscoff *et al.* [38] provided an upper bound to the number of contacts necessary to achieve form closure grasps. They showed that four contact points are sufficient for the form-closure grasp of any planar object and seven contact points are sufficient in spatial case. Bicchi [4] considered form closure as a purely geometric property of a set of contact constraints. Rimon and Burdick [59] gave precise definitions for first and second order form closure for frictionless grasps based on mobility theory. They also showed that a frictionless grasp is force closure if and only if it is form closure for both first order and second order.

2.4 Force Closure

To ensure that the object is grasped securely, the classical force closure condition is employed. A grasp of an object achieves force closure when it can resist any external wrench exerted on the grasped object. The well-known qualitative test for a force closure grasp is to check whether the contact wrenches of the grasp positively span the whole wrench space [61]. This is equivalent to checking whether the convex hull of the primitive contact wrenches contains the origin [40]. Various approaches for testing whether the origin is inside the convex hull are proposed. Yun-Hui Liu [35] proposed a recursive reduction technique which allows the problem of testing convex hull containing the origin in high dimensions to be solved in the lowest dimension. The same authors transformed this problem to ray-shooting which can be solved by linear programming [36]. Zhu and Wang [77] developed the force closure test based on the concept of Q distance which uses a convex hull containing the origin as a metric to test whether the origin lies in the interior of the convex hull of the primitive wrenches. Recently, Zhu *et al.* [76] discussed that the problem can be transformed into the problem of calculation of distance between convex objects. They proposed the use of pseudodistance function to solve the problem.

Other approaches of qualitative test for a force closure grasp by considering the workspace, not the wrench space, were also investigated. Nguyen [45] proposed a geometric method for testing two-finger force closure grasps on polygonal objects. The synthesis of stable grasps was proven by constructing virtual springs at the contact points, such that a desired stiffness matrix about its stable equilibrium can be acquired. Ponce *et al.* proposed the concept of non-marginal equilibrium which implies the force closure property. Based on this concept, the qualitative tests of three-finger grasps for polygonal objects [56] and four-finger grasps for polyhedral objects [57] were proposed.

For regrasping, a set of force closure grasps has to be calculated. In [56] and [57], a grasp is represented by parameters related to positions on the grasped faces. To calculate all possible grasps, two(three) additional parameters are required to construct linear constraints for two(three)-dimensional case. The additional parameters have to be eliminated to acquire a set of force closure grasping positions on given grasped faces. Sudsang and Ponce [67] proposed another representation of grasps avoiding the use of additional parameters. A point in workspace is used to represent a set of force closure grasps.

Quantitative tests of force closure grasps are also considered to define the quality of grasps. Kirkpatric *et al.* [30] considered the most general stability measurement which does not know a priori knowledge of disturbance. An external wrench is assumed to be uniformly distributed in every direction. The minimum magnitude of a particular external wrench that breaks force closure property is measured. This is equivalent to the radius of the maximal ball that can fit inside the convex hull of primitive contact wrenches. Ferrari and Canny [23] applied this criterion to plan the optimal grasp. The radius of maximal ball is used in many works, such as [5, 28, 39].

Recently, the best performance in resisting external wrenches as the optimality criterion is still studied. Yun-Hui Lui [36] addressed the problem of minimizing the L_1 norm of the grasp forces in balancing an external wrench, which can be transformed to ray-shooting problem. Zhu and Wang [77] addressed the problem of planning optimal grasps that minimize the Q distance and expresses the best performance in firmly holding an object while resisting external wrench loads. Zhu *et al.* [76] solved the same problem by optimizing the pseudodistance function.

Methods mentioned above are used to determine grasps that require precision of fingertip on the objects. To allow some positioning errors, the notion of *independent contact regions* was introduced by Nguyen [45]. In short, an independent contact region is a parallel-axis rectangular region in fingers' configuration space which represents areas on object's boundary where fingers can be placed independently to compose a force closure grasp. In [45], Nguyen also showed how to geometrically determine independent contact regions for two-fingered grasps of a polygon. Tung and Kak [69] attacked the completeness of the previous work and proposed an algorithm which is correct and complete. Recently, Cornella and Suarez investigated an algorithm of determining independent grasp regions on two-dimensional discrete objects [15]. A four frictionless grasp is considered. The algorithm determines the independent regions of two fingers when the locations of the other two fingers are given.

In order to find the *best* independent contact region, one needs to define what *best* means. There have been many different definitions of the best independent contact region due to different purposes and constraints of grasping devices. The two popular criteria are: (1) the largest n-cube, and (2) the largest rectangular region (product of lengths on every axis). Using the first criterion, the optimization can be done by linear programming as discussed in [56] and [57]. Faverjon and Ponce [21] tackled the problem of two-fingered grasping on curved objects using the second criterion. In their work, a numerical optimization algorithm was presented, but they could not guarantee the algorithm's completeness. Cornella and Suarez [16] presented an approach to determine independent contact regions on polygonal objects considering arbitrary number of friction or frictionless contacts on given edges. Their approach subdivides configuration space so that the graspable region in each subdivision is convex, then computes the independent contact region in each subdivision.

2.5 Regrasp Planning

Regrasp or dexterous manipulation is required when a grasp is not appropriate for a specific task. A planner calculating a sequence of feasible configuration of robot hand and object transforming to the desired one is applied to solve the problem. The obtained results from a planner have to satisfy constraints considered in the system. The distinction between various planners are constraints discussed above, kinematics, dynamic, stability constraints, etc. In this work, force closure constraint is satisfied only for more general results. Some different planners are discussed here.

Hong *et al.* [25] proved the existence of two and three finger grasps for two- and three-dimensional objects assuming isolated hard point contacts with friction. The manipulated objects are assumed to be smooth. This paper also proposed a fine motion of an object by repositioning the grasping fingers while maintaining a grasp during entire process. A subclass of fine motion problem focused in this paper is gait problem. Finger gaits with three and four fingers on the plane are proven for the existence. For the prove of three finger gait, a two finger force closure condition is taken into consideration. In the case of four finger gait, two different gaits can behave which are using two pairs separately or using a three finger grasp and replacing one finger with the remaining finger to form a new grasp.

Regrasp planning for reorientation of a prism was addressed by Omata and Nagata [49]. The four-fingered hand and frictional contact point are assumed. The planner plans a sequence of repositioning of fingers for horizontal rotation of an object for a desired angle. The calculation of finger repositioning are classified into three problems. Problem $A(c)$ tests whether the finger c can be removed from the initial grasp. This problem can be solved by linear programming method. Problem $B(c, n)$ is solved for calculating feasible region of finger c to form equilibrium grasp without finger n. The last one is problem $C(c, n, d)$ which calculates the feasible region of finger c when finger c and n form a grasp without finger d. These two problem can be solved by non-linear programming. Problem C is harder and takes more calculation time than B. Sequences of finger repositioning are attained by a search tree. Each node represents a removed finger. The search algorithm begins with solving problem $A(c)$ then solves $B(c, n)$ to remove finger n and bring finger c to form a grasp. Problem C will be solved when the problem B cannot produce feasible solution. Child nodes are expanded according to a heuristic function. The function is based on a angle which a grasp can rotate the object, the depth of a node and the penalty when problem C has to be solved.

Omata and Farooqi [50] studied object reorientation by using regrasp primitive. Two primitives are carried out for reorientation task. The *rotation* presented in [49] is a primitive that the fingers grasp on the side faces of the object and rotate it. The *pivoting* primitive uses the two fingertips to form an axis of pivoting and the third finger exerts the force on the side facts to rotate the object about the axis. The algorithm of this primitive is explained in this paper. Based on the following assumptions, four fingered hand and a prism object, sequential executions of these primitives can achieve reorientation. The search tree is applied to solve the problem. Each branch represents a primitive and each node contains the current orientation. The search procedure uses quaternion concept to solve resultant rotation about a unique axis.

An approach to solve the problem of dexterous manipulation using geometrical reasoning techniques was proposed by Munoz *et al.* [43]. Kinematic constraints are respected by checking non-penetration between the fingertips and the object. Some accessibility limitations due to the kinematic constraints of the hand are also considered. Three manipulation modes, which are fixed-point, rolling and sliding, are applied in the planning algorithm. A combination of manipulations in these three modes can form a nominal trajectory of a task that the object is being grasped by a dexterous hand. A manipulation task is represented by a homogeneous transformation that brings the object from its initial configuration to its final configuration. The planner decomposes the transformation into a sequence of infinitesimal motions by exploring the space of potential solutions for the problem of changing the orientation of the grasped object. Each infinitesimal motion is solved for every manipulation mode. The equilibrium constraints are considered in this procedure. A solution is represented in the form of joint motion. The minimum joint motion is selected by the planner for the particular infinitesimal motion.

In [11], The system of Cherif and Gupta assumed that the manipulation system processes at low velocities. Planning feasible quasi-static trajectories for the fingertips to move object to a desired configuration is available. Two motions which are rolling and sliding the fingertips on the surface of the object are considered. The planner is a two-level planning scheme. The global planning level applies an A^* search algorithm to find connectivity between sub-goals in the configuration space of the object. The nominal path generated by this planner ignores any manipulation constraints. The second level is the local manipulation planner. The local planner is based on solving an *inverse finger motion problem* to plan for feasible quasi-static motions of the hand-object system between sub-goals. The instantaneous solution satisfies collision-free, reachability, friction and equilibrium constraints.

Han and Trinkle [24] proposed a Framework for dextrous manipulation by rolling fingers on the surface of an object and finger gaiting. Three taxonomies of manipulation tasks for multifingered hand systems are stated: *Object Manipulation* , *Grasp Adjustment* and *Dextrous Manipulation*. The contribution of this paper is to purpose a general methodology to implement large-scale object manipulation tasks when the capability of the fingers are limited by their workspace. Two strategies, *finger rewind* and *finger substitution*, are applied to accomplish a task. Dextrous manipulation of a sphere is exemplified. The condition of two soft-finger and three hard-finger force closure grasp are derived for spherical object. The trajectory of the finger on the object is restricted to be a great circle which simplifies contact constraint.

2.6 Manipulation Planning

Since an object cannot move by itself. The robot hand has to grasp and move it from one stable position to another. The objective of the planner is to calculate a path of robot hand and object's configuration from an initial configuration to a desired configuration while avoiding collision with obstacles, other objects and self-collision.

Modelling the problem with as fully dynamic and using control-based planning is costly expensive. Thus, Alami *et al.* [1] developed another approach using two distinct paths which are transfer paths and transit paths. The former are defined as motions of the system while the robot hand grasps the object. Transit paths are defined as motions of the robot when it moves alone while the object is in a stable position. Regrasping operation is also calculated by the planner. Based on this concept, Koga and Latombe [31] solved the manipulation problem for robots with many degrees of freedom. The planner compute a series of transfer and transit paths for the robot that make the robot grasp and move the object from an initial configuration to a goal configuration. Recently, probabilistic algorithms are applied for manipulation planner under continuous grasps and placements in [63], [60].

Nielsen and Kavraki [46] developed a manipulation planner which extends the probabilistic roadmap (PRM) frameworks. The planner consists of two levels. The first level builds a manipulation graph. Nodes represent stable placements of the object. Edges represent transfer and transit actions. The actual motion planning for the transfer and transit paths is done by PRM planners at the second level. The fuzzy roadmap was introduced to apply in both levels. The computations is efficient by verifying that the edges are collision-free only if they are part of the final path. Instead, the local planner assigns a probability to the edge that expresses its belief that the edge is collision-free.

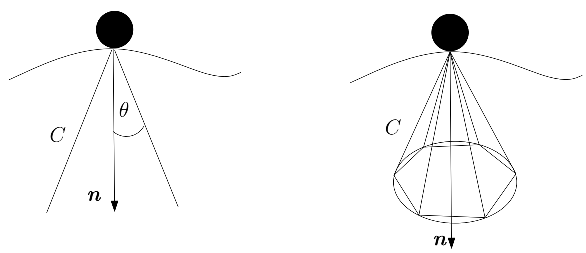
3 Background

Background knowledge according to regrasp planning consists of three main sections. In Section 3.1 relation between grasping and friction will be discussed. Equilibrium and force closure property are also described in Section 3.2. And a representation of a grasp inducing a grasping set on the surface of polygon and polyhedron is presented in Section 3.3.

3.1 Contact Model

In grasping, the most commonly used contact model are hard contact without friction, hard contact with friction and soft finger contact. Soft contact grasp is different from hard contact grasp with ability that soft finger can exert torque about the surface normal while hard finger can exert force at contact point only. For analysis of hard contact, the point contact without friction can only exert a unidirectional force normal to the surface. Tangential forces can be produced by a finger up to the friction coefficient when friction is considered.

Coulomb friction [66] is usually applied for friction model. Coulomb's law of friction states that for a contact point exerting a force f_N along the contact normal, the friction force (the tangential contact force) is less than or equal to $f_t = \mu f_N$ where μ is the frictional coefficient. This equation indicates that when the contact is maintained without slip, the contact can exert any force in a cone C of which the half angle is equal to $\tan^{-1}(\mu)$. The cone is emanated from the contact point and the axis coincides with the contact normal n. This cone is commonly called a friction cone. Cone in two-dimensional case can be expressed by two vectors as shown in Figure 1(a). In three-dimensional case, a cone is described by quadratic function. Cone introduces complexity of nonlinearity to the problem. To simplify the problem, a cone can be replaced with an m-sided pyramid (Figure 1(b)). A pyramid has planar facets which avoid nonlinearity from the problem but at a price of lesser precision.



(a) (b)

Figure 1: Coulomb friction: (a) is the friction cone for two-dimensional grasps and (b) is the friction cone for three-dimensional grasps and its approximating pyramid cone.

3.2 Equilibrium and Force Closure

In this section, some necessary backgrounds on force closure grasp are are given. We restrict our attention to hard fingers and assume Coulomb friction. A hard finger in contact with some object at a point $\boldsymbol{x}$ exerts a force $\boldsymbol{f}$ with moment $\boldsymbol{x} \times \boldsymbol{f}$ with respect to the origin (but it cannot exert a pure torque). Force and moment are combined into a zero-pitch wrench $\boldsymbol{w} = (\boldsymbol{f}, \boldsymbol{x} \times \boldsymbol{f})$. Under Coulomb friction, the set of wrenches that can be applied by the finger is:

$$W = \{(\boldsymbol{f}, \boldsymbol{x} \times \boldsymbol{f}) : \boldsymbol{f} \in C\}, \tag{1}$$

where C denotes the friction cone at $\boldsymbol{x}$. Combining force and torque into wrench makes it simpler to consider force closure property. An effect of a contact point or external disturbance can be easily described as a wrench.

A d-finger grasp is defined geometrically by the position $\boldsymbol{x}_i (i = 1, ..., d)$ of the fingers on the boundary of the grasped object. We can associate with each grasp the set of wrenches $W \subset \Re^k$ that can be exerted by the fingers where $k = 3$ for two-dimensional grasps and $k = 6$ for three-dimensional grasps. If we denote by W_i the wrench set associated with the i^{th} finger, we have

$$W = \{\sum_{i=1}^{d} \boldsymbol{w}_i : \boldsymbol{w}_i \in W_i \text{ for } i = 1, ..., d\}. \tag{2}$$

Let us firstly consider an equilibrium in the aspect of wrenches. An object is said to be in equilibrium when the summation of all force and torque acting on the object is zero, i.e., the net acting wrench is a zero vector.

Definition 3.1 (Equilibrium) *Let $\boldsymbol{w}_1, \ldots, \boldsymbol{w}_n$ be the wrenches acting on an object. We say that the object is in equilibrium when $\sum_{i=1}^{n} \boldsymbol{w}_i = \boldsymbol{0}$.*

We say that a grasp achieves force closure when any external wrench can be balanced by wrenches at the fingertips. We assume that a hand can exert infinite wrench. The object being grasped is also regarded as an ideal rigid body without deformation. Under this assumption, size of a wrench is neglected. Only the direction of wrench is considered. A force closure is defined by a scaling invariant property called *positively span* which is a property of a set of vector.

Definition 3.2 (Positively Span) *We say that a set of k-dimension vector $\boldsymbol{w}_1, \ldots, \boldsymbol{w}_n$ positively spans $\Re^k$ when any vector in $\Re^k$ can be represented by a positive combination[1] of $\boldsymbol{w}_1, \ldots, \boldsymbol{w}_n$.*

The relation between positively span and closure property is described by Salisbury [61]. Although force closure can be naturally described in terms of positively spanning, it is not clear how to identify whether a set of vectors actually positively span the space. However, the definition of positively spanning is equivalent to the definition of a convex hull and it is shown in [41] that a set of vectors $\boldsymbol{V}$ positively span a space when the origin of the space lies strictly inside the convex hull of $\boldsymbol{V}$.

Definition 3.3 *A set of wrenches W in $\Re^k$ is said to achieves force closure when the origin lies in the interior of the convex hull of W.*

According to the above definition, because zero wrench is contained in W for a force closure grasp, it is then clear that force closure implies equilibrium.

Since W is produced from forces in friction cones, a wrench can be described in the form of the wrench cone associated with a friction cone. In two-dimensional case, a friction cone can be defined by a positive combination of two vectors that defines the edges of the cone. This means that W can be described as a set of positive combinations of two wrenches associated with these two vectors. In three-dimensional case, the boundary of a cone is defined by quadratic equation. This introduces nonlinearity to the problem. A cone is approximated by an m-sided pyramid of which boundary is represented by m vectors to avoid nonlinearity. A positive combination of wrenches associated with these m vectors is used instead of the actual W.

Since we can consider only the directions of forces and wrenches, a force at a boundary of the friction cone at contact point is assumed to be a unit force. Wrenches associated with the forces bounding

[1] linear combination with positive coefficients

the friction cones at contact points are called *primitive wrenches*. The force closure property for a grasp can be described in term of primitive wrench. Let a hand be in contact with an object at the points $\boldsymbol{p}_1, \ldots, \boldsymbol{p}_n$. Force exerted at a contact point is assumed to lie in m-sided friction cone. The total wrench acting on the object can be expressed as convex combination of primitive wrenches by

$$\boldsymbol{w} = \sum_{i=1}^{n} \sum_{j=1}^{m} \alpha_{i,j} \boldsymbol{w}_{i,j} \tag{3}$$

and $\alpha_{i,j} \geq 0, \sum_{i=1}^{n} \sum_{j=1}^{m} \alpha_{i,j} = 1$.

A definition of force closure grasp can be formally stated in term of primitive wrenches as follow.

Definition 3.4 (Force Closure Grasp) *We say that a grasp making n contact points with an object achieves force closure when its associated primitive wrenches $\boldsymbol{w}_{1,1}, \ldots, \boldsymbol{w}_{n,m}$ achieves force closure.*

According to Definition 3.3, a grasp has to check whether the convex hull of primitive wrenches contains the origin in its interior. Let us firstly consider a relaxed condition which test whether the origin is a member of the convex hull. The condition can be formulated into the following constraints.

$$\begin{aligned} \boldsymbol{o} = \sum_{i=1}^{n} \sum_{j=1}^{m} \alpha_{i,j} \boldsymbol{w}_{i,j} \\ \sum_{i=1}^{n} \sum_{j=1}^{m} \alpha_{i,j} = 1 \\ \alpha_{i,j} \geq 0 \end{aligned} \tag{4}$$

where o is the origin.

The constraints give the way to compute convex combination of primitive wrenches, which produces any vectors in the convex hull. If we can find values of $\alpha_{i,j}$ subject to all constraints, this means that the origin is a member of the convex hull. However, this is just a necessary condition for force closure property since it has to ensure that the origin strictly lies in the interior of the convex hull. Fortunately, in computation, value of half angle can be reduced a little bit from exact value. This causes the origin which ever lied in the boundary of a reduced convex hull, strictly lying in the interior of the exact convex hull. Grasps such that the forces achieving equilibrium strictly lying inside the friction cones at the fingertips are called *non-marginal equilibrium grasps*. In two-dimensional cases, it is formally shown in [45] for two finger cases and generalized to three finger cases in citePoFa93 that a sufficient condition for force closure is non-marginal equilibrium grasps. For three-dimensional cases, Ponce *et al.* [57] proved that non-marginal equilibrium grasps achieve force closure.

3.3 Representing a Grasping Point and Grasping Set

In this work, we consider three types of object models which are contact points set, polygon and polyhedron. In case of contact points set, position of a contact point is given w.r.t. the data acquisition frame and can be directly used in computation or easily transformed to another frame. In the other models, representation of a grasping point need more detail to describe a contact point.

We assume that the model of an object is given w.r.t the object frame. The grasping point of finger i on an edge of a polygon is represented by a parameter u_i. A point $\boldsymbol{x}_i$ on a line segment is calculated by

$$\boldsymbol{x}_i = \boldsymbol{s}_i + u_i \boldsymbol{d}_i$$

subject to $0 \leq u_i \leq l_i$ where $\boldsymbol{s}_i$ is the starting point of the line segment, $\boldsymbol{d}_i$ is its normalized direction and l_i is its length. For a given edge, the normal vector pointing in the interior of the object is also given. The direction of normal force and normalized forces bounding the friction cone is pre-calculated to be constant vectors $\boldsymbol{f}_{\boldsymbol{O},1}, \ldots, \boldsymbol{f}_{\boldsymbol{O},m}$. Primitive wrenches at grasping point $\boldsymbol{x}_i$ are $(\boldsymbol{f}_{\boldsymbol{O},j}, \boldsymbol{x}_i \times \boldsymbol{f}_{\boldsymbol{O},j})$.

To represent the grasping point of finger i on a face of a polyhedron, a local coordinate frame $\{\lambda_1^i, \lambda_2^i, \lambda_3^i\}$ is attached to the face. The origin of the frame is located at any point lying in the same plane of the face. The coordinate is defined according to the right-hand rule, the λ_3^i axis is parallel to the normal of the face. The grasping point is represented by the local coordinates (u_i, v_i) on $\{\lambda_1^i, \lambda_2^i\}$ plane. We denote $\boldsymbol{o}_\lambda^i$ and $\mathbf{R}_\lambda^i$ as the origin and the rotation matrix of the local frame w.r.t. the object frame, respectively. The coordinates of the grasping point $\boldsymbol{x}_i$ w.r.t. the object frame are calculated by

$$\boldsymbol{x}_i = \boldsymbol{o}_\lambda^i + \mathbf{R}_\lambda^i (u_i \quad v_i \quad 0)^T$$

The faces of a polyhedron are assumed to be convex polygon. The parameters u_i and v_i representing grasping points on the face are therefore subject to linear constraints bounding the face w.r.t the local frame.

Similar to case of polygon, the normal of the given face introduces the constant normalized forces bounding the friction cone which is linearized to m-sided friction cone. And the primitive wrenches can be calculated in the same manner.

We are now ready to describe about *grasping set*. A grasping set is the feasible region of parameters representing grasping points on the given polygon edges or polyhedron faces, which achieve force closure property , i.e., grasping points satisfy the constraints in (4). In two dimensions, according to the number of parameters using in the representation of grasping points, a grasping set for n edges is subset of n-dimensional space. Grasping a polyhedron, a grasping set of n given faces is subset of $2n$-dimensional space.

4 Switching Graph for Polyhedral-modelled Objects

Regrasp is a process of repositioning contact points of robot fingers. Two primitive forms of repositioning are *finger switching* and *finger sliding*. To determine an appropriate sequence of these two processes, we introduce a structure called a switching graph. A node in a switching graph represents a connected set of force closure grasps on three(four) particular polygonal edges(faces) in two(three) dimensions. An edge connecting two nodes indicates that there exist a grasp associated with one

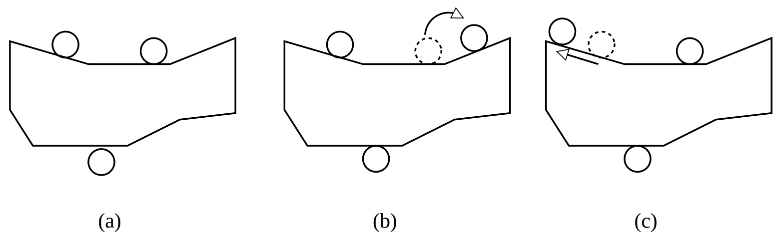


Figure 2: (a) Initial grasping configuration (b) A result of finger Switching. (c) a result of a finger sliding

node that can be switched to a grasp associated with the other by finger switching. By using a switching graph, the regrasp problem can be formulated into a graph search problem. A path from the graph search determines a sequence of actions – switching and sliding to be executed in order to traverse from the initial to the final grasp. We have already proposed switching graph to solve regrasp planning problem for polygon [53, 54] and polyhedron [55]. The following sections will describe the finger switching and sliding primitives and the switching graph in detail.

4.1 Finger Switching and Finger Sliding

Regrasp process which changes grasping configuration by placing an additional finger on desired contact point and then releasing one finger of the initial grasp is called finger switching. For example, let us assume that a starting grasp holds a polygonal object on points a, b and c and we want to switch to a grasp holding points b, c and d. A finger switching process starts by placing an additional finger on d and then releasing the finger at a. If both grasps satisfy the force closure property, the entire process still holds the force closure property. For the case of four(five)-fingered hand grasping a polygonal(polyhedral) object, finger switching requires that two(three) grasping configurations must have two contact points in common and both of them achieve force closure.

Finger sliding is a process for repositioning fingers by sliding them along edges(faces) of a polygon(polyhedron) while maintaining a force closure grasp during the sliding process. Using this process, we can change grasping configuration with in the same set of force closure grasps. This means the relation between finger sliding and a node of switching graph. However, finger sliding may be hard to implement mechanically since it is required that fingers must always touch the edge during sliding. Finger switching can imitate finger sliding by switching fingers from the initial to the final position of the sliding. Examples of finger switching and sliding are shown in Figure 2.

4.2 Node of a Switching Graph

A node in a switching graph represents a connected set of force closure grasps. In two-dimensional grasps, grasping with three fingers is sufficient while four-fingered grasps are considered in three dimension. These induce a set of force closure grasps for three(four) particular edges(faces) is a subset of three(eight)-dimensional configuration space according to the number of parameters using to represent a grasping point. A node in switching graph for given surfaces is associated with a connected component of grasping set. We use the constraints in Equation 4 to directly determine nodes corresponding to a particular combination of edges(faces). One grasping configuration can be

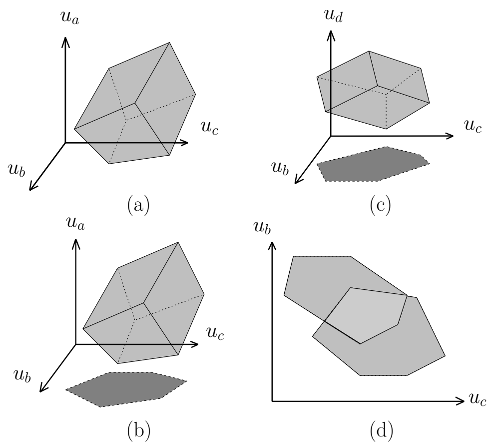


Figure 3: (a) A connected grasping set representing possible grasping points (in term of u_a, u_b and u_c (b), (c) two sets and their projections. (d) Intersection of the projected subspace representing a set of common points for a finger switching.

changed to another by finger sliding when two of them are in the same node of the graph because of continuity in a connected set of force closure grasps.

4.3 Edge of a Switching Graph

An edge linking two nodes, v_x and v_y, indicates that a finger switching can be performed between a grasping configuration in v_x and the other in v_y. Finger switching requires that two(three) non-switching contact points on polygon edges(polyhedron faces) must remain the same during the process. It follows that there will be an edge connecting two nodes when there are two grasping configurations, each of which belongs to each node, that use the same grasping points on the non-switching polygonal edges. Formally, in two dimensions, there will be an edge connecting a node v_x and a node v_y when there exist a triple of points (p_a, p_b, p_c) on edges a, b, c in the grasping set of v_x and a triple (p_a, p'_b, p'_c) on edges a, b, c in the grasping set of v_y such that $p_b = p'_b$ and $p_c = p'_c$. For three-dimensional finger switching, three common contact points are required and the existence of finger switching can be stated in the same way of two-dimensional cases.

To demonstrate the existence of finger switching, in two dimensions, let P_1 be a connected grasping set for edges $\{a, b, c\}$ and P_2 be a connected grasping set for edges $\{b, c, d\}$. The space of P_1 and P_2 have two components (axes) in common, namely the axes of u_b and u_c. These components correspond to the non-switching edges, i.e., the common edges of both grasps. The projection of P_1 on the space of these two components represent the set of points on edges b and c that a force closure grasp on a, b and c is possible. Similarly, the projection of P_2 represents a set of points for a force closure grasp on b, c and d. If the intersection between these two projections is not empty, then there exist points on b and c that form a force closure grasp on both a, b, c and b, c, d. The reverse is also definitely true. Figure 3 depicts the projection process.

5 Switching Graph for Discrete Contact Point Set

Many researches about regrasping exploit continuity of surface for regrasping with finger sliding or rolling when the model of object is given such as polygon, polyhedron or algebraic surface. In contrast, regrasping between two grasps on discrete contact points is clearly related to finger switching process. Finger sliding and rolling cannot be explicitly applied when the information about local surface is unknown. The problem of regrasping on discrete contact point set is stated as follows.

Consider a set of contact points $P = \{\boldsymbol{p}_1, \ldots, \boldsymbol{p}_n\}$ sampled from the surface of an object and equipped with unit normals $N = \{\boldsymbol{n}_1, \ldots, \boldsymbol{n}_n\}$ directing into the interior of the object. In practice, points P and their unit normals N are generated from mechanical probes scanning the surface through physical contact. However, this method is not appropriate for scanning soft materials or hard accessible objects such as concave objects. Non-intrusive techniques are therefore more popular. One interesting technique is *range scanning*, i.e., the distance from the scanning device to the sample points on the surface is estimated using optical triangulation, interferometric technique using laser light or time to flight principle. In this case, a set of points are given from range scans and normals are estimated from range data during the space acquisition phase or by local least-square fitting. Assuming that P and N have been already given, our method will calculate a switching graph over the set of contact points. A node of the graph represents a force closure grasp with three and four contact points in two and three dimensional grasps, respectively. Each triple or four contact points are tested whether they satisfy the force closure property. If the condition is achieved, a node representing the grasp exists. An edge between two nodes is easily defined. If the grasps of two nodes have two(three) common contact points for two(three)- dimensional grasps, there exist an edge joining them.

At a glance, we can imagine the complexity of a naive switching graph's construction. The computation of all nodes takes time $O(n^3)$ and $O(n^4)$ in two and three dimensions. For all edges, it takes $O(n^6)$ and $O(n^8)$, respectively. An occurring challenge is that a method handling a large number of contact points is required. Some algorithms reducing the complexity of computing force closure grasps from a set of contact points are proposed [47,48]. These algorithms can be directly applied in the computation of nodes in the graph. However, reducing the complexity of the construction of a graph and its structure is still a problem to be overwhelmed.

Another problem is exploiting the information of the grasped surface. Without local surface information, finger sliding along the surface cannot be performed. Local properties of contact points can be acquired according to model fitting selected for estimating the surfaces. One standard used for the representation of complex geometric objects is polygonal meshes especially triangular meshes. Triangular meshes are preferred due to their algorithmic simplicity, numerical robustness, and efficient display. The advantage of this representation is that algorithms for triangular meshes usually work for shapes with arbitrary topology and algebraic structures are not the severe polynomial patches. Another advantage of using this representation is that it can be used for many stages of the typical processing pipeline in geometric design applications without the need for inter-stage data conversion. This causes the reduction of all processing time and error in the conversion. The triangular mesh model has been commonly used in many works [2, 10, 17, 26, 32, 64, 65, 70]. However, a conversion to higher order surfaces is needed when the information about the surface topology is not sufficient for further processing. In this case other representations take place or the triangular surface is converted to higher order meshes such as subdivision surfaces, spline surface etc. Remeshing the triangular mesh with higher order meshes, Bajaj *et. al.* [3] uses Bernstein-Bézier polynomial implicit patches to refine a representation of the surface, that are guaranteed to be smooth and single-sheeted. The refinement using B-spline fitting are shown in [20, 71]. The construction of non-uniform rational B-Splines (NURBS) surface patch network are proposed by Park *et. al.* [51, 52].

Exploiting surface information, a path between any two points on the surface can be calculated. For finger sliding, we consider two different grasps which have more than one distinct contact positions(one different contact point can be handled by finger switching). A path between a couple of different points on the surfaces is calculated and every step along the path is verified for force closure property. There are various ways to test the paths, for examples, vary one grasping position while the others are fixed or grasping positions are concurrently moved. If the paths are feasible, there exist an edge joining two nodes representing these two grasps.

6 Objective of the Work

- To propose a framework and develop an efficient algorithm that solves the regrasp planning problem based on a switching graph structure from a polyhedral-modelled object and a set of finite contact points.

7 Research Methodology

7.1 Preliminary Works

- Study related works in grasping and dexterous manipulation as discussed in Section 2.
- Propose the switching graph structure which allows our planner to search a path from a grasping configuration to a desired one. In case of polyhedral-modelled object, a node in switching graph represents a set of force-closure grasps for given surfaces and an edge joining two nodes indicates a set of grasping configurations for initial surfaces, which can be switched to a set of grasps for other surfaces having one different surface from the initial surfaces.
- Apply sufficient conditions for force closure grasp for the construction of switching graph for polygon and polyhedron. In two dimensions, a sufficient condition for three-fingered concurrent grasp is applied. However, the condition is strongly restricted. Necessary and sufficient conditions for two-fingered grasp and three-fingered parallel grasp are employed to reduce incompleteness. In three dimensions, a sufficient condition for four-fingered concurrent grasp is used. Since the algorithms manipulating polyhedron are complex therefore an randomized approach is proposed to reduced the running time.

7.2 Ongoing Works

- Study necessary and sufficient conditions for force closure grasp. In [56], two-dimensional three-fingered force closure grasps are classified in two types which are concurrent grasp and parallel grasp. In [57], Grassman geometry [19], which characterizes the varieties of various dimensions formed by sets of dependent lines, can be applied to yield a necessary and sufficient condition for non-coplanar four-fingered force closure grasps in three dimensions. Force-closure grasps are classified into concurrent grasps, pencil grasps and regulus grasps. To avoid the taxonomy of grasps, a condition of force closure grasp which checks whether the convex hull of primitive wrenches contains the origin, can be applied instead.

- Develop efficient regrasp planning algorithms for polyhedral-modelled object and discrete contact point set.
- Publish a journal article relating to the works.
- Write the thesis.

8 Scope of the Work

This work proposes a framework for regrasp planning in both two and three dimensions. The framework acts upon the second level of the abstraction in Table 1. Switching graph is our framework for solving this problem. A node in the graph represents a connected set of force closure grasps which can be calculated according to any formulated grasping condition. A set of grasping configurations which can switch between any two grasps in two different connected grasping sets indicates an edge joining the associated nodes. An obvious advantage of this framework is that the obtained solution is a set of feasible grasping configurations meeting only grasping constraints. This means that the solution can be further used by applying task requirements or mechanical constraints. Moreover, a task planner can determine a set of desired grasping configurations independently with the lower constraints based on the abstraction. Another advantage which can be exploited is graph structure. This allows regrasp planning to be transformed to graph search problem.

To apply a switching graph, this work proposes efficient algorithms dealing with sets of force closure grasp for polygon and polyhedron. Three- and four-fingered force closure grasping conditions are used in two and three dimensional cases, respectively. One additional finger is used for finger switching. Thus, robot hands are assumed to be equipped with four and five fingers. To gain set of general solutions, robot fingers are assumed to be free-flying points in the workspace. In a case of discrete point set, contact points and their normals are assumed to be obtained from approximated triangular meshes. A point on each triangular surface is picked to be a contact point and the normal vector of the surface directing into the object is the normal of this contact point.

9 Contribution of the Proposed Work

- We gain a framework and efficient algorithms working on regrasp planning problem in two- and three-dimensional workspace where the inputs are polygon, polyhedron or discrete point set.

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