Computation of Force Closure Grasps from Finite Contact Point Set

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1 Introduction

A Robot, from its very name, literally means "forced labor". To live up to this title, an ability to interact with milieu is crucial. A robot should be able to manipulate objects. With a hand capable of grasping, a robot is blessed with greater degree of influent. Complex manipulation and maneuver are possible with grasping. Moreover, grasping allows robot to use tools readily available for human. For example, instead of having a robot with various switchable specialized end effectors, we can build a robot with dexterous hands and make available tools usable by both robot and human. The later situation is more robust than the first. Without a doubt, grasping is an essential ability of a robot, especially a humanoid one.

Colloquially, grasping means firmly holding of an object. There are many definitions of a firm grasp, either geometrically or mechanically. Many of grasping definitions, even though defined disparately, are eventually shown to be equivalent. The study of these definitions constitutes the ground theory of grasping. The essence of these definitions is the same: an object being grasped should be secured by contact points of a robot, even under a presence of reasonable external disturbance[1]. For example, a grasping of a hammer is considered firmly when the hammer can be effectively used, i.e., the hammer does not fall off from the hand due to the gravity and the hammer does not fly off the hand when it is swung and hit against nails. The hammer should still be in the hand of the robot even someone forcefully tries to take it away. These properties informally define a firm grasp.

A firm grasp is not hard to achieved. It has been shown that, with a sufficient number of contact points, a firm grasp is always possible on any object [22]. An algorithm for synthesizing a grasp is also easy, i.e., taking linear time under very relaxed constraints [26]. However, not all grasps are born equal. In a simulated world where many issues are considered negligible, all grasps are considered the same but when a real practical grasping hand is considered, many constraints arise such that some grasps are more preferable than the others. This discrimination originates from the fact that the act of grasping can naturally be separated into two layers, the task aspect and the hand aspect. The task aspect indicates a meaning or an objective of grasping, i.e., it indicates a use of grasps. The hand aspect imposes constraints on a geometrical process of grasping, e.g., certain grasp is not possible

[1] Another variation of grasping is "caging", where an object being caged is not fixed but is bounded in a finite region. Caging, to some extent, is considered a weak form of grasping and have an extensive study in its own right

with some particular hand. Situated between this two layers is a grasp planning. Grasp planning acts as a consolidator between these two layers. It determines a grasp that is suitable to both aspects.

Layering helps decompose the problem. It allows a hardware practitioner to concentrate on creating a hand. At the same time, robot visionary can imagine any use of robot in manipulation. The problem is that, currently, we have too many of tasks and hands. As we have stated before, grasping is quite a fundamental action. Its use comes in a broad range and there exist lots of robotic hands in the present. In the literature, a common approach is to choose a specific task and/or a specific hand and identify goodness evaluator that suits them. After that, an appropriate algorithm is tailored such that a good performance is possible. Obviously, that algorithm works best on that particular setting but the performance deteriorates when setting is different.

Currently, there is no algorithm that works best on any setting. This is mainly because grasping tasks are varying where a particular task may require a property that is a trade-off with the other. For example, let us consider grasping of a hammer. If we are to relocate a hammer, it is best to grasp it on its metal head to reduce the effect of moment. However, if we wish to hammer something, the hammer should be grasped on its handle so that the moment of its head is maximized. Additionally, there is no universally accepted model of a hand. Different hands come with different constraints. A priori knowledge of a task and a hand is necessary for a design of an appropriate algorithm.

This problem of task/hand performance dependency is the inspiration of our work. We aim to derive an algorithm that is applicable with any task or hand in the real world. The key concept is simple; a task and a hand impose on grasping algorithms many constraints, which we simply choose to neglect them. We believe that an algorithm of grasp planning should be separated from a grasping process. Without any assumption on a task or a hand, all grasps are treated without any prejudice. If a grasp satisfies the definition of firm grasp, it should be gracefully considered as a fully qualified grasp. Instead of grading and reporting just the best grasp, the algorithm should report as many solutions as possible. Solutions from the algorithm will be dependent only on the object being grasped, not with any predefined hand or task. After solutions are identified, when knowledge of a hand and a task is provided, we then prioritize solutions accordingly. This can be done in linear time since all solutions are already computed. The same solution set can be reused with other combination of a hand and a task also.

Providing multiple solutions is also beneficial to regrasping problem. Sometime, a grasp is needed to be modified, i.e., contact positions is needed to be changed while a grasp should still firmly hold an object. The act of changing a grasping configuration while maintaining a firm grasp is called regrasping. With multiple grasping solutions at hand, we can identify a grasp to be changed easily.

Still, the problem of identifying all possible grasps without biased task/hand constraints is not challenging enough. Most works in the literature assume some geometrical model of the object being grasped with a linear model being at the apex of popularity. This is mainly because a linear model allows efficient or analytical formulation for characterizing grasps on a given object. Many works propose algorithms that do not consider the issue of selecting a grasping facet, they aim solely on deriving a grasping algorithm that works on a set of polygonal faces whose number is equal to the number of contact points. When the number of faces of the object exceeds the number of contact points, an exhaustive search is performed. By assuming that the object can be modeled with a polygon with minuscule number of faces, the algorithm should works acceptably. There are only a few works that bear the goal of polygonal face selection as a main objective, much less the works that applicable on the object as a whole.

Obviously, linear model cannot accurately represent every real world object. In practice, to apply such methods, we have to sense an object and then the sensed data has to be approximated by a polygon. With a limited number of edges in the polygonal model, there are always some objects (such as curved objects) that may not fit well. The resulting inaccurate model could then lead to unreliable resulting grasps. Of course, modeling accuracy can be improved simply by using more polygonal edges. This action, however, increases the model's complexity which results in higher computational cost in selecting combination of edges for which good grasps can be found. An alternative that addresses this issue assumes the curved object model in computing grasps. Although a curved model may better represents the shape of some objects, the cost of model fitting and grasp computation are significantly higher than using polygonal models. Not many works in the literature derive a method for curved objects.

In this work, a different approach is pursued. Our method does not operate on any boundary model; the input of the method is a set of points on the object's boundary together with the corresponding contact normals. Clearly, with our approach, there is no need for fitting the sensed object with a specific boundary model. Objects in any shape can therefore be handled in the same manner with the same accuracy. Nevertheless, from a practical standpoint, it is legitimate to ask about the performance of the proposed approach. To accurately model an object, a large number of boundary points is required. The number of points might be as many as a hundred to a few thousand points. This poses a great challenge to the problem, considering that a brute force algorithm would take time in the order of $O(n^k)$ where k is the required number of contact points. A work that does not consider the issue of face selection obviously suffers a severe performance problem. However, this approach will result in a solution with much greater accuracy and it should be applicable in the real world.

The obvious problem of this approach is that we will be overwhelmed with too many grasps. Constraints imposed by a task or a hand provide a way to eliminate second-rated solutions. Without them, we have to consider every grasps. It is of crucial importance that we can compute solutions with blazing fast speed. This is the goal of our algorithm: to compute as many grasps as possible and as fast as possible under modeless assumption of the object being grasped.

In conclusion, we aim to create an efficient algorithm that reports as many firm grasps as possible, regardless of any other quality criteria. Unlike the majority of the literature, no explicit model of the object being grasped is assumed. The algorithm takes as input the set of boundary points and their corresponding normal. In the next section, an in-depth review of grasping literature is presented along with the formal definitions of concepts which are conversationally discussed in this section, such as firm grasp, quality criteria, etc.

1.1 Problem Statement

Given a set of n contact points, each of which is described by a position vector $\boldsymbol{r}_i$ and an inward contact normal vector $\boldsymbol{n}_i$, we have to compute several force closure grasps.

2 Reviews on Grasping

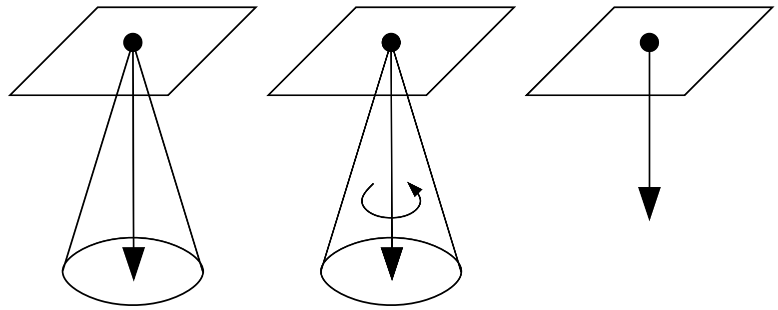
Works on grasping and fixturing received lots of attention during the last two decades. Many works were published since the pioneering work of Salisbury and Roth [44]. This section review some of relating works. Interested readers should refer to $[2, 27, 32]$ for more references of the works.

2.1 Hands and Contact Models

The problem of hand dependency stems from the fact that there is no universally-accepted model of a hand. Obviously, the diversity of hands is beneficial. Different task requires different design of hand. The more a robotic hand is designed to resemble the actual human hand, the more complex it is. Only a few robotic hands are available at the present, due to high cost and sophisticated manufacturing process required for building a robotic hand. Let us review some of well known robotic hands in this section. A very simple hand, yet widely used, is a parallel jaw gripper. The gripper comes with two parallel plates which can squeeze into each other. A parallel jaw gripper allows a simple form of grasping. Nevertheless, it lacks capability to grasp a complex object. Another robotic hand that is also widely available is a Barrett Hand [50], commercially made by Barrett Technology Inc. Barrett Hand comes with three fingers, two of which can be spread synchronously by 180° around the palm. Another well-known hand is the DLR-Hand [6] designed and developed at German Aerospace Center (DLR). The hand consists of four fingers that resembles the human hand but the size is noticeably larger. The Robonaut Hand [20] designed by NASA's Johnson Space Center consists of five fingers and the size is the same as a very big male hand. The hand looks more like the actual human hand than the DLR-hand.

To consider a hand analytically, one has to choose how to model a contact between a hand (fingers) and an object. Two main aspects have to be taken into account: whether a finger is hard or soft and whether friction exists. A hard finger implies that a contact is modeled as a point while a soft finger allows a face contact. Friction governs how force can be exerted by a contact. With friction, a hard contact can exert not only a force along its normal but also forces inside a cone defined by the frictional coefficient. A soft finger with friction can exert pure torque in addition to forces exertable by a hard finger. Figure 1 shows various contact models.

Friction is usually represented by the Coulomb's model [46]. Coulomb's law of friction states that for a contact point exerting a force f_N along the contact normal, the friction force (the tangential contact force) is less than or equal to $f_t = \mu f_N$ where μ is the frictional coefficient. This equation indicates that when the contact does not slip, the contact can exert any force in a cone of which the half angle is equal to $\tan^{-1}(\mu)$. The apex of the cone locates at the contact point and the axis coincides with the contact normal. This cone is commonly called a friction cone. Cone in 2D can be expressed by two vectors. However, in 3D case, a cone cannot be described by any linear function. Cone introduces complexity of nonlinearity to the problem. Many works choose to simplify this problem by replacing a cone with an m-sided pyramid. A pyramid has planar facets which abolish nonlinearity from the problem but at a price of lesser precision. Unless m is large enough, the accuracy of the problem is reduced significantly.



(a) (b) (c)

Figure 1: Contact Model: (a) Hard finger with friction. Force can be applied inside a cone defined by the frictional coefficient. (b) Soft finger with friction, additional pure torque can be exerted by the contact. (c) Hard finger without friction, force can be applied only along the normal direction

2.2 Grasping Definition

The concept of a firm grasp is formalized in various ways. The concept is considered in both geometrical and mechanical points of view. Grasping properties that are commonly used are *equilibrium*, *form closure* and *force closure*. A grasp is in equilibrium when the resultant of applied forces and torques are zero. Force closure indicates that the grasp can exert a resisting force and torque that balance any external disturbance on the object. Form closure is closely related to the force closure and they are confusingly used in place of each other. Usually, form closure speaks of the immobility of an object in presence of fixed contact points. The motion of the object is prevented by the positioning of contact points. Form closure considers the problem in geometrical point of view [3]. It does not incorporate the capability of contact point. This is different from force closure which considers how contact points can exert force and torque on an object. Directions of forces that contact points can apply on an object determines force closure property. Form closure is shown to be equivalent to force closure when no friction is taken into account [27, 28, 30]. Form closure is considered as a stronger variation of force closure, i.e., every form closure grasp is also a force closure. That is the reason why form and force closure is used interchangeably in the literature, especially in the frictionless case. However, in their essence, the concept is different in the perspective from which the problem is analyzed. An evident distinction between form and force closure is the presence of friction [3]. Friction effect is considered in force closure while it is neglected in form closure analysis.

Equilibrium, force closure and form closure are the properties determined by a configuration of contact points, i.e., they do not consider other properties of a grasped object except the position and the normal direction of the contact points. Without contact, a hand cannot interact with an object and thus these properties cannot be achieved. [2]

Recently, Rimon and Burdick presented another definition of firm grasp called second order immobility [40–42]. Second order immobility indicates that a grasp can prevent finite motion but not the infinitesimal one. There exists a non force closure grasp that achieves second order immobility. Second order immobility, in a sense, ensure a firm grasp of an object since the object is confined in an infinitesimal configuration space. Unlike the discussed grasping properties, second order immobility

[2] This is different from caging which does not require contact between a hand and an object. Caging is not considered in this works. Interesting reader should confer, for example, to [39, 47, 48] for more information about caging.

relies on curvature information at contact positions. This forbid our current setting, which take into account only the position and normal direction of contact points, from second order immobility.

In this work, we emphasize on force closure grasp. As stated before, a grasp achieves force closure when it can balance any external force and torque acting on the object being grasped. When the grasped object is disturbed by some external force or torque, the contact points must exert appropriate force and torque to counter it. This requires an accurate measurement of the external disturbance and good processing power to compute online balancing force. When comparing with the form closure property which immobilizes an object geometrically, it seems that force closure is harder to implement. However, since form closure does not deal with friction, it requires more number of contact points, as will be shown in the upcoming section. A lesser number of contact points makes force closure more promising in many works. This also reduces complexity of a robotic hand. Moreover, it seems that a human hand follows the notion of force closure. For example, a human tends to produce counter force when the grasped object is disturbed, e.g., being pulled away or hitting with other object. Adopting force closure in our works should benefit to a more human type of grasping.

2.3 Force Closure

Before the discussion of force closure, let us formalize how kinetic entities are described. Force closure deals with force and torque. Force is written as a vector and so is torque. These two variables are denoted by two separate variables. For unity, these two are usually modeled as one vector entity called a *wrench*. A wrench is a vector of force concatenated with a vector of torque. In 2D space, force can be described by a 2D vector while torque is described by a 1D vector, so, a wrench in 2D space is a 3D vector. Likewise, a wrench in 3D space is 6D vector formed by a 3D force vector concatenated with a 3D torque vector. Formally, a wrench $\boldsymbol{w}$ is denoted by $(\boldsymbol{f}, \boldsymbol{t})$ where $\boldsymbol{f}$ is a force vector and $\boldsymbol{t}$ is a torque vector. Combining force and torque into wrench makes it simpler to consider force closure property. An effect of a contact point or external disturbance can be easily described as a wrench.

Let us firstly consider an equilibrium in the aspect of wrenches. An object is said to be under equilibrium when the summation of all force and torque acting on the object is zero, i.e., the net acting wrench is a zero vector.

Definition 2.1 (Equilibrium) *Let $\boldsymbol{w}_1, \ldots, \boldsymbol{w}_n$ be the wrenches acting on an object. We say that the object is in equilibrium when $\Sigma \boldsymbol{w} = \mathbf{0}$.*

Force closure property indicates that a grasp can exert a wrench that can balance any other wrench. Normally, only the direction of wrench is considered. It is assumed that a hand can exert infinite degree of wrench. The object being grasped is also regarded as an ideal rigid body without deformation. Under this assumption, size of a wrench is neglected. A force closure is defined by a scaling invariant property called *positively span* which is a property of a set of vector.

Definition 2.2 (Positively Span) *We say that a set of n-dimension vector $\boldsymbol{w}_1, \ldots, \boldsymbol{w}_n$ positively spans $\mathbb{R}^n$ when any vector in $\mathbb{R}^n$ can be represented by a positive combination[3] of $\boldsymbol{w}_1, \ldots, \boldsymbol{w}_n$.*

[3] linear combination with positive coefficients

Hereafter, we say that a set of vectors positively span $\mathbb{R}^n$ when they satisfies Definition 2.2. Salisbury [43] describes the relation between positively span and closure property. Although force closure can be naturally described in terms of positively spanning, it is not clear how to identify whether a set of vectors actually positively span the space. However, the definition of positively spanning congruences with the definition of a convex hull and it is shown in [26] that a set of vectors $\boldsymbol{V}$ positively span a space when the origin of the space lies strictly inside the convex hull of $\boldsymbol{V}$.

Definition 2.3 *A set of wrenches W in $\mathbb{R}^n$ is said to achieves force closure when the origin lies in the interior of the convex hull of W.*

Up until this point, we have described grasping properties using wrench notation. Next, we will define how a grasp achieves these grasping properties. When a hand comes into contact with an object at various contact points, the hand can influent the object via the contact points. The effect of the hand can be described in term of wrenches. According to the contact point model, one contact point might be able to exert only one unique wrench or a wrench in a wrench set defined by a friction cone. Assume that a hand exerts a force $\boldsymbol{f}$ at a contact point $\boldsymbol{p}$ with normal direction $\boldsymbol{n}$. The effect of the hand can be described in terms of a wrench $\boldsymbol{w}$ as follows.

$$\boldsymbol{w} = (\boldsymbol{f}, \boldsymbol{p} \times \boldsymbol{f} + \boldsymbol{\tau}) \tag{1}$$

A pure torque modifier $\boldsymbol{\tau}$ can only be exerted when a soft frictional contact model is assumed. When a contact is hard and frictionless, $\boldsymbol{f}$ has the same direction as $\boldsymbol{n}$. Under frictional case, a hand can exert a force inside a cone as described in Section 2.1. Let θ denotes the half angle of a friction cone. A hand can exert a set of wrench as follows.

$$\mathcal{W} = \{(\boldsymbol{f}, \boldsymbol{p} \times \boldsymbol{f} + \boldsymbol{\tau}) | \hat{\boldsymbol{f}} \cdot \hat{\boldsymbol{n}} \leq \cos(\theta)\} \tag{2}$$

Obviously, $\mathcal{W}$ consists of wrenches from forces in a friction cone. In 2D, a friction cone can be defined by a positive combination of two vectors: $\boldsymbol{f}_l$ and $\boldsymbol{f}_r$ that defines the edges of the cone (see figure 2). This means that $\mathcal{W}$ can be described as a set of positive combinations of two wrenches associated with $\boldsymbol{f}_l$ and $\boldsymbol{f}_r$. However, in 3D, there exists an infinite number of vectors that defines boundary of a cone. A wrench set $\mathcal{W}$ is actually defined according to (2). This introduces nonlinearity to the problem. It is common that a cone is approximated by an m-sided pyramid of which boundary is represented by m vectors. A positive combination of wrenches associated with these m vectors is used, as an approximation, in place of the actual $\mathcal{W}$.

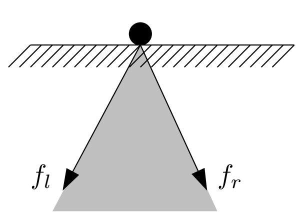


Figure 2: 2D friction cone

Since force closure does not consider the magnitude of a wrench, a wrench $\boldsymbol{w}$ exerted by a contact point is assumed to be a unit wrench. Normalized wrenches that can be exerted by a contact are usually called a *primitive contact wrenches*. Unless stated otherwise, any wrench from this point onward is assumed to be a unit wrench for the sake of clarity.

We are now ready to define the force closure property for a grasp. Let a hand be in contact with an object at the points $\boldsymbol{c}_1, \ldots, \boldsymbol{c}_n$. Let $\mathcal{W}_i$ be a set of primitive contact wrenches associated with $\boldsymbol{c}_i$.

Definition 2.4 (Force Closure Grasp) *We say that a grasp making n contact points with an object achieves force closure when its associated primitive contact wrenches set $\mathcal{W}_n, \ldots, \mathcal{W}_n$ achieves force closure.*

Obviously, the value of a wrenches varies according to the choice of the origin. When the reference frame is changed, a torque component of a wrench is also changed. However, a force closure property is invariant to the choice of the origin.

Proposition 2.5 *Let a grasp makes n contact points at $\boldsymbol{p}_1, \ldots, \boldsymbol{p}_n$ with force $\boldsymbol{f}_1, \ldots, \boldsymbol{f}_n$. When the grasp achieves force closure w.r.t. the reference frame, it is also achieve force closure w.r.t. the any arbitrary frame A described by a Euclidian transformation.*

Proof: Let T be the Euclidian transformation constructed from a rotation R and a translation $\boldsymbol{t}$ that describes a frame A w.r.t. the reference frame. Let $\boldsymbol{w}_i = (\boldsymbol{f}_i, \boldsymbol{p}_i \times \boldsymbol{f}_i)$ be the wrench associated with i^{th} finger. Let $\boldsymbol{w}'_i$ be the wrench associated with i^{th} finger described in the frame A, i.e., $\boldsymbol{w}'_i = (R\boldsymbol{f}_i, (R\boldsymbol{p}_i + \boldsymbol{t}) \times R\boldsymbol{f}_i)$. We will show that any arbitrary vector $\boldsymbol{v} = (\boldsymbol{f}_v, \boldsymbol{m}_v)$ can be described by a positive combination of $\boldsymbol{w}'_1, \ldots, \boldsymbol{w}'_n$.

Let $\boldsymbol{q} = R^{-1}\boldsymbol{v} = (R^{-1}\boldsymbol{f}_v, R^{-1}\boldsymbol{m}_v)$. Since $\boldsymbol{w}_1, \ldots, \boldsymbol{w}_n$ achieve force closure, there exist positive values $a_1, \ldots, a_n$ such that $a_1\boldsymbol{w}_1, \ldots, a_n\boldsymbol{w}_n = \boldsymbol{q}$. By multiplying a_i to $\boldsymbol{w}'_i$, we would obtain $a_1\boldsymbol{w}'_1, \ldots, a_n\boldsymbol{w}'_n = (R(R^{-1}\boldsymbol{f}_v), \boldsymbol{m}) = (\boldsymbol{f}_v, \boldsymbol{m})$. The value of $\boldsymbol{m}$ is equal to

$$
\begin{aligned}
\Sigma\left(a_i(R\boldsymbol{p}_i + \boldsymbol{t}) \times R\boldsymbol{f}_i\right) &= \Sigma(a_i R\boldsymbol{p}_i \times R\boldsymbol{f}_i) + \Sigma(a_i t \times R\boldsymbol{f}_i) &(3)\\
&= R\Sigma(a_i\boldsymbol{p}_i \times \boldsymbol{f}_i) + \Sigma(a_i t \times R\boldsymbol{f}_i) &(4)\\
&= R(R^{-1}\boldsymbol{m}_v) + \Sigma\left(a_i t \times R\boldsymbol{f}_i\right) &(5)
\end{aligned}
$$

This means that $a_1\boldsymbol{w}'_1, \ldots, a_n\boldsymbol{w}'_n = (\boldsymbol{f}_v, \boldsymbol{m}_v + \Sigma(a_i t \times R\boldsymbol{f}_i))$. Obviously, there exists a solution $b_1, \ldots, b_n$ such that $b_1\boldsymbol{w}_1, \ldots, b_n\boldsymbol{w}_n = (0, -R^{-1}\Sigma(a_i t \times R\boldsymbol{f}_i))$. Since the summation of force part of $\Sigma b_i\boldsymbol{w}_i$ is zero, when we multiply b_i to w'_i, the value of $\Sigma\left(b_i t \times R\boldsymbol{f}_i\right)$ would be zero. Thus, $\Sigma b_i\boldsymbol{w}_i$ is equal to $(0, -\Sigma(a_i t \times R\boldsymbol{f}_i))$. This means that $\boldsymbol{v}$ can be described by $(a_1 + b_1)w'_1, \ldots, (a_n + b_n)w'_n$. ■

Friction plays an important role in force closure. Most notably, Nguyen shows that, in 2D two-fingered grasp, a subset of equilibrium grasp called non-marginal equilibrium also achieves force closure. Ponce and Faverjon proved the same concept for three fingers [34]. This implication is also true in 3D case as shown in [35], also by Ponce *et al.* Non-marginal equilibrium is an equilibrium in frictional model where forces achieving equilibrium are lying not on the boundary of their respective friction cone. This is equivalent to substitute a less-than sign ($<$) for a less-than-or-equal sign ($\leq$) in (2). In practice, it means that any equilibrium grasp is also a force closure grasp under any arbitrarily greater frictional coefficient.

2.3.1 Existence of Grasp

Definition 2.2 and 2.3 provide qualitative tests (yes/no) whether a particular grasp achieves force closure. However, they cannot be used to determine whether an object can be grasped in a force closure manner. Early works of grasping investigate on the bound of the number of contact points required for satisfying various grasping properties.

The earliest works on form closure is by Reuleaux [37] (reprinted in [38]) who shows that at least four contact points are required for form closure in 2D. Lakshminarayana [15], citing the work of Somov [45], reported that at least seven contact points are needed in 3D case. The result is confirmed by Markenscoff, Ni and Papadimitriou, who proved in [22] that there is no set of n vectors that positively span $\mathbb{R}^n$. In frictionless setting, this indicates that we need at least four (seven) contact points to achieve form closure in 2D (respectively 3D).

Upper bounds on the number of end effectors are investigated in [26] by Mishra, Schwartz and Sharir. It is shown, by using Carathéodory's theorem and Steinitz's theorem, that there exists upper bounds of the number of contact points that can always achieve form closure for piecewise smooth objects, with some exception to a particular class of objects (circular or rotational symmetric object) which cannot be grasped by any number of contact point. In 2D, an object can be grasped with equilibrium and with form closure by four and six contact points, respectively. The required number of contact points increases to seven and twelve in 3D case. This marks a loose bound on the number of required contact points.

The bound is tighten by Markenscoff, Ni and Papadimitriou in [22]. They show that, without considering the exception class of the problematic objects in [26], form closure can always be achieved by four wrenches in 2D and can be achieved by seven wrenches in 3D. This bridges the gap between the upper bounds and the lower bounds presented in [15].

For frictional case, Markenscoff, Ni and Papadimitriou also show that force closure can always be achieved by three and four contact points for 2D and 3D case, respectively. Under non-zero friction, force closure always exists for any piecewise smooth objects without any exception. These include circles and rotational symmetric objects which are problematic in the form closure case. Unlike frictionless case, the lower bounds for frictional case are not the same with the upper bound. Nguyen shows in [29] that it is possible to construct a force closure grasp using two contact points in both 2D and 3D cases.

2.4 Grasping Qualities

Although we aim to treat all grasps without quantitative measurement, it is worth considering some existing performance indices of a grasp. Most performance measurement can be categorized into two broad groups: those that value stability of a grasp and those that value grasping accuracy. The stability of a grasp is, informally, how well a grasp can withstand external disturbance. For accuracy, two issues are considered. The first one is how severe the error of contact point positioning affects configuration of the object. The second one is how well a grasp tolerate positioning error.

The most general stability measurement does not take a priori knowledge of disturbance, i.e., it assumes that an external wrench is uniformly distributed in every direction. Intuitively speaking, it measures the minimum magnitude of a particular external wrench that breaks force closure property, given a fixed grasping force. In the wrench space, this is equivalent to the radius of a maximal ball that can fit inside the convex hull of primitive contact wrenches. This measurement is introduced by Kirkpatric, Mishra, Yap [14] and popularized by Ferrari and Canny [9]. However, this measurement is not invariant to the choice of the origin since wrench contains torque which dependes on the choice of origin.

The frame dependence of this measurement is pointed out by Li and Sastry in [17]. They suggest the use of the volume of the primitive contact wrench space instead. Still, the radius of maximal ball is used in many works, such as [4, 12, 25].

Another stability measurement is the magnitude of wrenches that achieve a firm grasp. Trinkle [51] proposes a measurement that measures how far the grasp is from losing form closure property. He uses the maximum of minimum value of the wrench magnitude that achieves equilibrium. Kerr and Roth [13] minimize the equilibrium forces applied by the contact point. Markenscoff and Papadimitriou [23] minimize the magnitude of force, under the worst case scenario that a grasp still achieve form closure under a unit external wrench.

Ponce and Faverjon propose an indirect measurement of stability. They minimize the distance between the centroid of an object and the center of mass of contact points [34]. In fact, they use L_∞ distance so that the problem is linear. The work is extended into 3D in [35]. Ding, Lui and Wang use quadratic programming (QP) to optimize the grasp under the same measurement in [8]. By utilizing a QP, they can use Euclidian distance, instead of L_∞ distance. Lesser distance between the centroid of an object and the center of mass of contact decreases the effect of gravitational and inertial force on the object.

Ding *et al* [7] choose contact points that minimize positioning error of an object when the contact points are misplaced. The objective of this minimization follows D-optimality criteria used by Wang in [53].

Nguyen [28] introduces the problem of finding a grasping region, (rather than a grasping point), that always achieve force closure. As long as contact points are placed on the region, force closure is guaranteed regardless of the exact position of contact point. This region is called *independent contact region*. Ponce and his colleagues [34, 36] derived algorithms that use the size of independent contact regions as a measurement. Independent contact region copes with the error of contact point placement. The larger the region is, the more the placement error is allowed while the force closure property is still satisfied.

Recently, Zhu and Wang [55] present the concept of *Q distance*. Given a convex polyhedron P, the Q distance indicates whether the origin lies inside, outside or on the surface of the polyhedron P. The Q distance consists of two sub-measures which are Q^+ distance and Q^- distance. Both of them are subject to a measuring polyhedron A. The polyhedron A can be any convex object that has the origin lying in its interior. In practice, A is defined to be a simplex with its centroid at the origin. The Q^+ distance is the smallest scaling factor of A such that $A \cap P$ is not empty. A positive value of the Q^+ distance indicates that P does not contain the origin. The Q^+ distance can be zero if and only if the origin is contained in P. The Q^+ distance is used to indicate whether the origin is in P but it can not indicate whether the origin lies strictly inside P. This is where the Q^- distance comes into plays. The Q^- distance is the largest scale factor of A such that $A \cap P = A$, i.e., the largest scaling factor for which A lies entirely in P. Obviously, if the origin lie on the boundary of P, the value of the Q^- distance must be zero. A positive value of the Q^- indicates that the origin lies in the interior of P. The value of Q is obviously subject to A. If we use a sphere as A, the Q^- distance becomes exactly the same measure of Ferrari and Canny [9]. However, Zhu and Wang suggest the use of a simplex as A since it requires the minimal calculation.

2.5 Conditions for a Force Closure Property

Early works in robotics grasping concentrates on defining what actually is a firm grasp (as reviewed in section 2.3.1). What the researchers came up are the ideas of force closure and form closure (and some other properties). Following that, various conditions of force and form closure are frequently proposed. A condition for force closure is usually represented as a qualitative test, i.e., it identifies whether a grasp achieves force closure. According to the definition, obvious qualitative tests of force closure is a test of positively spanning and a test whether the origin is inside the convex hull. Other tests are usually based on these definitions. Sometime, the problem is casted as a geometrical problem and so is the test.

Force closure requires that lines of action of the forces must be linearly dependent. Grassmann [24] categorizes the possibilities that lines are linearly dependent into various groups: coplanar lines, intersecting lines, a regulus and two flat pencils having a line in common. Of all these groups, the grasp with intersecting lines is the most common in the literature. This is popularized by Ponce and his colleague as follows. Ponce and Faverjon present a condition for 2D frictional force closure for two and three fingers [34], also with the different colleagues, Ponce presents a geometrical condition for four fingers in 3D [35]. Informally, it is shown that force closure is achieved when there exists forces lying inside friction cone that positively span and intersect at the same point. They also present a weaker condition for force closure that can be described linearly so that a linear programming can be employed to solve the problem. Recently, Li *et al.* [16] present a very simple geometrical condition for force closure in intersecting cases. The condition can be applied only for three fingers in both 2D and 3D cases. To the best of our knowledge, this is the current fastest test.

A geometrical approach usually does not consider all groups of Grassmannian geometry since they are so different that a unified condition would be too complicated. A more general force closure condition on grasping is usually considered in terms of wrenches where distinction between configuration of line of force is irrelevant. Following propositions formalize well known conditions for a set of vectors to positively span 2D and 3D space.

Lemma 2.6 *We say that a set of three vectors $\boldsymbol{v}_1, \boldsymbol{v}_2$ and $\boldsymbol{v}_3$ positively span the plane when the negative of any of these vectors lies strictly inside the cone formed by the other two vectors.*

Proof: *Sufficient condition:* Assume that $-\boldsymbol{v}_1$ lies strictly inside the cone formed by $\boldsymbol{v}_2$ and $\boldsymbol{v}_3$. Obviously, $\boldsymbol{v}_2$ and $\boldsymbol{v}_3$ lies on the opposite side of a line passing through $-\boldsymbol{v}_1$. This means that $\boldsymbol{v}_1$ also form a cone with $\boldsymbol{v}_2$ and also form a cone with $\boldsymbol{v}_3$. These two cones are non-intersecting against each other and also non-intersecting against the cone formed by $\boldsymbol{v}_2$ and $\boldsymbol{v}_3$. Thus, any point in the plane can be represented by one of these cones.

Necessary condition: Assume that the three vectors positively span the plane but the negative of one of them, says $\boldsymbol{v}_e$, does not lie inside the cone formed by the others. This means that the other two vectors must lie on the same side of the line formed by $\boldsymbol{v}_e$. Obviously, any point lying on the opposite side of the line cannot be represented by any positive combination of $\boldsymbol{v}_1, \ldots, \boldsymbol{v}_3$. Thus, the three vectors do not positively span the plane. ■

Fig. 3 shows the example of Lemma 2.6. This proposition can be easily extended to cover 3D cases as follows.

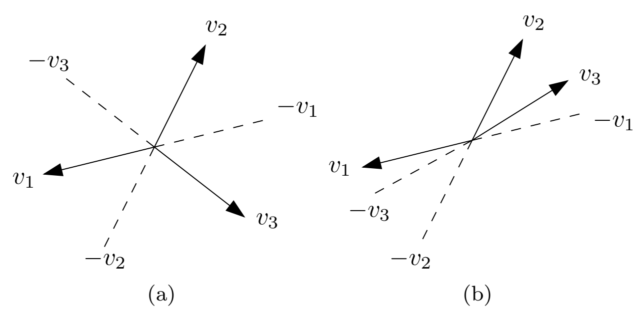


Figure 3: (a) Three vectors satisfying Lemma 2.6. The dashed lines represent the negative of vector. (b) Three vectors does not satisfy Lemma 2.6

Lemma 2.7 *We say that a quartet consisting of $\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3$ and $\boldsymbol{v}_4$ positively spans $\mathbb{R}^3$ when the negative of any of these vectors lies strictly inside the cone formed by the other three vectors.*

Proof: *Sufficient condition:* Again, let us assume that $-\boldsymbol{v}_1$ lies strictly inside the cone formed by $\boldsymbol{v}_2, \ldots, \boldsymbol{v}_4$. We will show that any point $\boldsymbol{x}$ can be represented by a positive combination of $\boldsymbol{v}_1, \ldots, \boldsymbol{v}_4$. If $\boldsymbol{x}$ lies inside the cone, it can be represented by a positive combination of $\boldsymbol{v}_2, \ldots, \boldsymbol{v}_4$. If it is not, then there exists a plane p such that p contains $\boldsymbol{x}$ and $\boldsymbol{v}_1$. Since $-\boldsymbol{v}_1$ lies inside the cone, p must also intersect with the cone. Let $\boldsymbol{v}_l$ and $\boldsymbol{v}_r$ be the edges of the cone that intersect with p. Vector $\boldsymbol{v}_l$ and $\boldsymbol{v}_r$ are positive combination of $\boldsymbol{v}_2, \ldots, \boldsymbol{v}_4$. Obviously, $\boldsymbol{v}_1, \boldsymbol{v}_l$ and $\boldsymbol{v}_r$ positively span on plane p. Thus, $\boldsymbol{x}$ can be represented by a positive combination of $\boldsymbol{v}_1, \boldsymbol{v}_l$ and $\boldsymbol{v}_r$. Which means that $\boldsymbol{x}$ is a positive combination of $\boldsymbol{v}_1, \ldots, \boldsymbol{v}_4$.

Necessary condition: Assume that the $\boldsymbol{v}_1, \ldots, \boldsymbol{v}_4$ positively span the space but none of them has its negative lies inside a cone formed by the others. This means that there exists a plane p defined by two vectors, let them be $\boldsymbol{v}_2$ and $\boldsymbol{v}_3$ without loss of generality. From the assumption, $-\boldsymbol{v}_1$ does not lies inside a cone formed by $\boldsymbol{v}_2, \boldsymbol{v}_3$ and $\boldsymbol{v}_4$. This means that $\boldsymbol{v}_4$ and $\boldsymbol{v}_1$ is on the same side of plane p. Obviously, any point lying on the opposite side of this plane cannot be represented by any positive combination of $\boldsymbol{v}_1, \ldots, \boldsymbol{v}_4$. Thus, they can not positively span the space. ■

Ding *et al.* provided a proof of a more general version of Lemma 2.7 which can be found in [8]. Fig. 4 illustrates four vectors that satisfy this lemma.

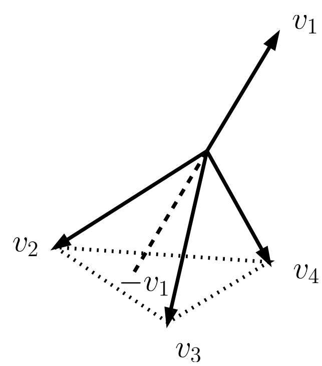


Figure 4: An example of four vectors satisfying Lemma 2.7

Mostly, the positively span problem is represented as an optimization problem which check whether there exists feasible solutions under constraints of contact position and friction. Sometimes, the problem is casted as a linear programming while other as a quadratic programming, for example, check [8].

The other test of force closure is a check whether the origin lies inside the convex hull of primitive contact wrenches. Usually, this approach constructs all primitive contact wrenches and then constructs their convex hull. After that, the origin is check whether it is inside. Since dimension of wrenches may be as high as six, a multidimensional algorithm for constructing a convex hull is needed. A very popular implementation of convex hull is Qhull [1]. However, naively constructing a convex hull and check for the origin is not efficient since we do not actually need the convex hull. Liu transforms the problem of the origin inside a convex hull into a ray shooting problem [18]. The new problem can be solved more efficiently by linear programming. Recently, Zhu *et al.* [54] discussed that the problem can also be transformed into the problem of calculation of distance between convex objects. They propose the use of pseudodistance function, such as GJK algorithm [10, 11], to solve the problem.

There also exists other conditions for a convex hull containing the origin. Liu [21] considered the problem, specifically in 2D grasping (which is 3D wrench space), in geometrical manner and derived a condition that reduces the wrench dimension of the problem by one. Ding *et al.* [8] also presented a condition that, for a given fixed set of wrenches, identifies whether an additional wrench positively span when combined with the given set.

2.6 Grasping with Modeless Input

Like our work, some researchers also recognize the problem of object model in grasping. Mostly, they assume that the input of the problem is a set of discrete point, rather than a modeled object either linear or nonlinear.

In the early work of Brost and Goldberg [5] and of Wallack and Canny [52], algorithms that deal with a discrete point set input are proposed. In these works, the fixturing positions are discretized but the object is not. Later, Wang [53] presented an greedy algorithm that works on real discrete point set. However, the proposed algorithm is not complete, i.e., it might not be able to find the best solution. Four years later, Liu *et al.* [19] present the algorithm for the same problem that is complete.

3 Objective of the Work

- To develop an efficient algorithm that reports several force closure grasps from a set of finite contact points.

Although the core concept of the problem is in mechanics, the very problem we are dealing with is an algorithmic problem. The knowledge of mechanics provides the fundamental understanding of the problem. After the problem is transformed into the study of positively spanning or origin inclusion, the intrinsic of the problem is rather mathematic. Many also consider the problem in terms of computational geometry. With our goal of reporting several solutions from a large input, our major concern is efficiency of the method. What we have to do is to derive a technique for speeding up the computation. Ineluctably, our problem is in fact algorithmic in nature.

An important point of a grasping problem is that there are multiple levels of the aspect of the problem. At the most primitive level, the problem requires understanding of mechanics. When we move up to a more abstract level, the problem is modeled as a mathematical problem. At our interested layer, the problem is obviously an algorithmic problem.

4 Research Methodology

Our preliminary work incorporates the goal of computing several solutions into a grasping method. By considering the sufficient condition for force closure in [34]. We could identify that part of the condition allows a use of heuristic that solve the problem much faster. We believe that there are many characteristic of various force closure conditions that, when consider with the objective of several solution in mind, can be exploit to have a faster algorithm.

Currently, the above believe is not only just a conjecture. A reformulation of force closure condition is recently studied [49]. The reformulation converts wrench space in 3D into 2D projective space. By using a projective space, we conjecture that many approaches can be used effectively to solve the problem. Our tentative solutions are tend to involve in this reformulation of problem.

4.1 Preliminary Works

- Study grasping properties
- Identify current trend in the literature and the current problem of task/hand dependency.
- Derive a heuristic algorithm that works faster in 3D case by utilizing several necessary conditions of the problem (published in [31]).
- In-depth study of grasp planning algorithms.
- Study related works in grasping (the regrasp problem, published in [33]).
- Perform extensive comparison of various grasping condition in both 2D and 3D.

4.2 Ongoing Works

- Study a reformulation of problem into a projective space [49].
- Develop a complete algorithm for 2D problem.
- Publish a journal article relating to the work.
- Prepare and engage in a thesis defense.

4.3 Benefit of the Work

- Having an algorithm that reports several force closure grasp from a set of discrete contact points.

5 Scope of the Work

- This work considers force closure grasping in both 2D and 3D.
- Derived algorithms must work faster than an enumerative approach that uses the fastest computation of force closure ([16] for 2D and [10] for 3D)
- Performance measurement can be either an actual running time (in case of a heuristic algorithms) or a complexity analysis (in case of a complete algorithms).

6 Benefit of the Work

We gain better algorithms working on a discrete point set. The algorithm is applicable regardless of a priori knowledge of a task at hand.

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